Permutation of the three-compartment equation

Diggory Hardy
Swiss TPH

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The three compartment equation:

$$C(t) = \sum_{i=1}^{n} D_i \left(Ae^{-\alpha(t-t_{D_i})} + Be^{-\beta(t-t_{D_i})} + Ce^{-\gamma(t-t_{D_i})} - (A+B+C)e^{-k_a(t-t_{D_i})} \right)$$

where $A, B, C, \alpha, \beta, \gamma, k_a$ are pharma-kinetic constants, $D_i (i \in \{1, ..., n\})$ are the the sizes of a series of doses given at times t_{D_i}, where $t_{D_i} < t$ for all i.

For simplicity, let us look only at the first component:

$$C_A(t) = \sum_{i=1}^{n} AD_i e^{-\alpha(t-t_{D_i})}$$

What is $C_A(t+1)$? Let us consider that additional doses D_i for $i \in \{n+1, ..., m\}$ may be given at times t_{D_i}. Thus we have:

$$\begin{aligned}
C_A(t+1) &= \sum_{i=1}^{n} AD_i e^{-\alpha(t+1-t_{D_i})} + \sum_{i=n+1}^{m} AD_i e^{-\alpha(t+1-t_{D_i})} \\
&= \sum_{i=1}^{n} \left[AD_i e^{-\alpha(t-t_{D_i})} e^{-\alpha} \right] + \sum_{i=n+1}^{m} AD_i e^{-\alpha(t+1-t_{D_i})} \\
&= \left[\sum_{i=1}^{n} AD_i e^{-\alpha(t-t_{D_i})} \right] e^{-\alpha} + \sum_{i=n+1}^{m} AD_i e^{-\alpha(t+1-t_{D_i})} \\
&= C_A(t) e^{-\alpha} + \sum_{i=n+1}^{m} AD_i e^{-\alpha(t+1-t_{D_i})}
\end{aligned}$$

We can thus write $C(t+1)$ as:

$$C(t+1) = C_A(t+1) + C_B(t+1) + C_C(t+1) + C_{ABC}(t+1)$$

$$C_A(t+1) = C_A(t)e^{-\alpha} + \sum_{i=n+1}^{m} AD_i e^{-\alpha(t+1-t_{D_i})}$$

$$C_B(t+1) = C_B(t)e^{-\beta} + \sum_{i=n+1}^{m} BD_i e^{-\beta(t+1-t_{D_i})}$$

$$C_C(t+1) = C_C(t)e^{-\gamma} + \sum_{i=n+1}^{m} CD_i e^{-\gamma(t+1-t_{D_i})}$$

$$C_{ABC}(t+1) = C_{ABC}(t)e^{-k_a} + \sum_{i=n+1}^{m} CD_i e^{-k_a(t+1-t_{D_i})}$$